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Type II small stringy black holes, probe branes and higher derivative interactions

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Abstract: The near horizon geometry of a fundamental string wrapped around an S^1 reduced to four dimensions is expected to be $AdS_2 \times S^2$. A probe string analysis suggests a no-force condition indicating supersymmetry, which coincides with the condition that the AdS_2 is embedded in AdS_3. We therefore consider the bulk string theory in terms of a WZW model on AdS_3 following recent proposals by Dabholkar et. al and Giveon et. al. We find that conformal symmetry of the model naturally leads to the no-force constraints obtained from the probes. Moreover, we are able to extract the values of the moduli that account for the value of the microscopic entropy. We also investigate higher derivative corrections of the form $\alpha'^3\mathcal{R}^4$ + flux terms to the horizon, in the context of type IIB supergravity. Imposing the no-force condition from the probe analysis leads to a striking simplification of the equations of motion at this order in α'. However, we argue that the value of the entropy can only be determined by considering all orders in α'.

Keywords: AdS-CFT Correspondence, Black Holes in String Theory.

JHEP03(2008)043

Contents

1. Introduction

Ideas concerning duality in the context of supergravity and string theory, have modified our understanding of classical and quantum aspects of black holes. In particular string theory has been successful in providing a correct microscopic description of the entropy of various classes of black holes [1], in accord with the Bekenstein-Hawking formula.

Several classes of multi-charged black holes in string theory have illustrated a match between microscopic and macroscopic entropies [1 − 3]. However there has been a puzzle regarding extremal black holes with two or less charges, which are usually referred to as small black holes. These have vanishing horizon area, and thus zero Bekenstein-Hawking entropy. In order to solve this apparent paradox, it is argued that the vanishing horizon can be traced back to the fact that the curvatures near the core of these black holes are very large, so that a horizon of the string scale could only be seen after including the higher derivative terms so far neglected [4]. This was shown explicitly in [5] for the heterotic winding string with momentum compactified to four or five dimensions. It was a remarkable coincidence that in this case the full entropy obtained from microscopic counting was reproduced by simply adding the Gauss-Bonnet term [6, 7] to the classical supergravity action.[1] The assumption behind this is that after supersymmetrizing this term (which has not been done), the related flux terms will not alter the conclusion. Higher order α' effects are also not expected to change the result although this has not been proved. It is then our main aim to look for the analogous statements for the type II fundamental string black holes.

In this paper we will study features of the two-charge black hole obtained by wrapping a type II fundamental string m times around a circle with momentum $p = n/R$, where R is its radius. The system is then compactified on T^5 to obtain a black hole in four dimensions. Such a configuration is supersymmetric and has been shown to have a non-zero entropy given by $S = 2\pi\sqrt{2nm}$. However, it is well known that the classical solution corresponding to such a black hole has vanishing horizon and therefore, zero Bekenstein-Hawking entropy. Hence, it is expected that higher α' corrections will radically alter the horizon geometry.

Since we do not have an explicit solution for a black hole with a stretched horizon in type II supergravity, we will follow Sen and assume that any extremal black hole must have a near-horizon geometry with an AdS_2 isometry. Clearly supersymmetry will provide further constraints on the geometry, but so far the α' corrected Killing spinor equations remain unknown. However, it is expected that appropiate probe strings and D-branes must experience no force. Since the background itself is supposed to be generated by a string wrapping a compact circle, the most natural probe one can use, is a string parallel to the background string. It is then our strategy to test the background with strings in order to determine a BPS condition relating the moduli (radii of the horizon metric and fluxes). Following the known giant graviton literature [9−11], we also considered dual probes which wind the sphere transverse to the background string, and tried to look for BPS-like conditions. We will find that these conditions restrict the black hole moduli in such a way, that the $AdS_2 \times S^1$ is enhanced to AdS_3.

The connection between AdS_3 and AdS_2 is familiar from the literature on five dimensional heterotic black holes (see [12, 13]). However, our probe analysis shows that the symmetry enhancement is also realised in the context of type II black holes, and furthermore motivates the possibility of looking at the fundamental string worldsheet theory being the holographic dual of this AdS_3 black hole geometry. This idea has been explored recently in the literature [14−17]. In particular in [14, 15] the bulk geometry is given by some $\mathrm{SL}(2, \mathcal{R})$ WZW model at level 2 which reproduces the symmetries of the fundamental string worldsheet CFT. Given that in this proposal the bulk geometry is an exact CFT,

[1] For $D > 5$ one needs to consider higher extended Gauss-Bonnet densities, which are of higher order in α'. This has been discussed in [8]

JHEP03(2008)043

one can read off the values of the moduli from the WZW action, and these values should be correct to all orders in α'. We will see that indeed, the WZW action implies the no-force conditions we found from the probe analysis, and that the values of the moduli reproduce the full entropy of the black hole, when they are substituted into the entropy function. The fact that one gets full agreement with microscopic counting, supports Sen's scaling argument for computing higher derivative corrections to the entropy. Furthermore, the no-force conditions are recovered from a particular WZW action describing strings in AdS_3 which supports the AdS_3/CFT_2 construction for the type II fundamental string.

We will then explicitly consider the effect of higher derivative corrections relative to the Einstein-Hilbert action, and investigate the effects of field redefinitions. In type II string theory, corrections to the supergravity action start at order α'^3, rather than α' as in the heterotic case. Also, unlike the heterotic case where the coefficient of the Gauss-Bonnet term is unity (with an appropriate normalization), the coefficient of the type II R^4 term involves an irrational $\zeta(3)$ factor. As a result, it is expected that the full entropy will not be reproduced in this case. In addition there are field redefinition ambiguities [18] at this order which could enter into the determination of the entropy. These arise from the fact that string scattering amplitudes can only determine the coefficients of the terms in the action that are not proportional to the lowest order equations of motion. As a result, the undetermined coefficients are ambiguous, and need to be fixed by other arguments, such as absence of ghosts [19] or "off-shell" supersymmetry [20]. These issues also enter in any kind of type II black hole setup. For instance, in [21] a particular class of field redefinitions was explored while considering corrections to the D1-D5p black hole, which has three charges and non-zero size. It was argued that the α'^3 corrections could be treated perturbatively, so that after linearizing the attractor equations, the entropy was free of any ambiguity. In [22], a single charge black hole was studied. In this case, the black hole has zero size and the α'^3 corrections cannot be treated perturbatively. Because of this, the authors chose a particular field redefinition in which the terms proportional to the Ricci tensor, were removed, and showed that the horizon was stretched.

We will investigate how the field redefinition parameters affect the entropy at order α'^3 and explore the special role played by the no-force conditions in distinguishing supersymmetric solutions from non-supersymmetric ones. We focus our attention on two cases. We first truncate the action and consider corrections coming only from the gravitational sector in the dimensionally reduced theory. Then we take into account the flux terms by using the conjecture in [23, 24]. We will find that restricting the corrections to the gravitational sector is in general inconsistent with the no-force conditions obtained from the probe analysis. However, once one includes contributions from the gauge fields, and imposes the no-force constraints, the attractor equations simplify drastically. This occurs independent of field redefinitions and allows one to solve for the moduli for each choice of field redefinition parameters. Yet, the entropy will depend on them, suggesting that for the type II small black hole, one needs to consider all α' corrections, in contrast to the heterotic case. This reinforces the need to treat the bulk geometry using the WZW exact CFT.

The plan of this paper is as follows. In section 2, we review the Kaluza-Klein compactification of type IIB supergravity, from ten to four dimensions, and we focus on the case of a

JHEP03(2008)043

winding string coupled to the Neveu-Schwarz field. In section 3, we briefly introduce Sen's entropy formalism for determining the entropy from the knowledge of the effective action, and apply it to our case of study. Section 4 presents a family of probe string and probe D2-brane solutions in the AdS background and the derivation of the no-force conditions. In section 5 we discuss the values of the moduli in the WZW model proposed in [14, 15] and the resulting entropy from Sen's entropy formalism. Finally section 6 is devoted to the computation and analysis of the higher derivative corrections to the entropy. Conclusions are then presented in section 7.

2. Type IIB action in four dimensions

The known supergravity solution of a BPS fundamental string winding a compact circle with some momentum p around the circle has, like many other extremal black brane solutions, vanishing horizon area [25]. However, albeit in a different regime in coupling constants, it is possible to do a microscopic counting of string states of the system and get a finite result for given winding number w and momentum p according to the Cardy's formula. It is suspected that by including g_s and α' corrections, to the supergravity Lagrangian, the horizon of the black hole becomes "stretched", so the area and subsequently the Bekenstein-Hawking entropy, is non-vanishing [4, 26 − 30]. A standard way for obtaining the corrections to the black hole entropy from higher correcting terms in the Lagrangian was devised in [5]. The basic assumption that underlies the arguments is that the geometry of an extremal black hole in D dimensions should be $AdS_2 \times S^{D-2}$ in the near horizon limit. To obtain a black hole for our system, namely a type IIB fundamental string winding around a compact circle with momentum, we can consider compactifying it on a five-torus and a circle i.e. $T^5 \times S^1$ to obtain a black hole in four-dimensions.[2] In the near horizon limit we should expect the metric to take the form [7]

$$ds^2 = a\left(-r^2 dt^2 + \frac{dr^2}{r^2}\right) + b d\Omega_2^2 \tag{2.1}$$

where a and b to be determined by extremalising the action after substituting in the corresponding metric. The system carries the electric charges of NS two form $B_{\mu\nu}$ and the Kaluza-Klein potential A_μ and we assume that at the horizon the field strengths of these potentials take constant values e_1 and e_2 respectively. The scalar fields including the (four dimensional) dilaton and the field parametrising the radius of the compact circle are all assumed to take constant values at the horizon.

$$e^{-2\phi} = u_s, \qquad e^{\frac{\psi}{2}} = u_T. \tag{2.2}$$

In order to evaluate f defined by [5]

$$f(\vec{u}, \vec{v}, \vec{e}) = \int_H d^H x \sqrt{-\det g} L, \tag{2.3}$$

[2]For heterotic fundamental string it is known that when considering four derivative corrections to the action, one obtains a black hole with a near horizon geometry of $AdS_2 \times S^2$ [31, 4]. For the type IIB case, no explicit solutions have been obtained.

JHEP03(2008)043

we need to reduce the ten-dimensional type IIB action to four dimensions. We are reducing the theory on $T^5 \times S^1$ and there are simple relationships between the higher dimensional fields and their counterparts after reductions. These relations[3] are given by [32]

$$\hat{G}_{\mu\nu} = \begin{pmatrix} g_{ab} + e^{\frac{\psi}{2}} A_a^2 A_b^2 & & g_{ty} \\ & R^2 \delta_{mn} & \\ g_{yt} & & e^{\psi} \end{pmatrix}, \tag{2.4}$$

where

$$\begin{aligned} \hat{G}_{tt} &= g_{tt} + e^{\psi}(e_2 r)^2 \\ \hat{G}_{ty} &= e^{\psi}(-e_2 r) \\ \hat{B}_{ty} &= -e_1 r \\ \hat{\phi} &= \phi + \frac{1}{4}\psi \end{aligned} \tag{2.5}$$

where the hat denotes the ten-dimensional fields. The lower case latin letters $a, b \in \{0,1,2,3\}$ and m, n runs from 4 to 8. y denotes the direction along the compact circle. R is the radius of the flat torus and is an arbitrary constant which could be absorbed in the definition of the four dimensional dilaton and conveniently set to one. The Kaluza-Klein potentials are given by $A_a = (-e_2 r, 0, 0, 0)$ near the horizon because we have assumed the field strength to be

$$F_{tr}^{(2)} = \partial_t A_r^{(2)} - \partial_r A_t^{(2)} = e_2. \tag{2.6}$$

The solution is static with no time dependence so we can set $A_r = 0$. Similarly we have

$$F_{tr}^1 = \hat{H}_{try} = e_1 = (d\hat{B})_{try}. \tag{2.7}$$

thus giving (2.5) above. Now we are ready to obtain the four dimensional reduced Lagrangian from the ten dimensional one. This is done by substituting the ten dimensional fields in terms of the four dimensional ones using (2.5). For example, the Ricci scalar in ten dimensions is reduced to

$$R^{(10)} = R^{(4)} - \frac{1}{4} e^{\psi} F_{(2)}^2. \tag{2.8}$$

The lowest order effective type IIB action in four dimensions in the NS sector is given by

$$S_{IIB(4)} = \frac{2\pi V_5}{16\pi G_{10}} \int d^4x \sqrt{-g_4}\, e^{-2\phi} \left[R^{(4)} - \frac{1}{4} e^{\psi} F_{(2)}^2 - \frac{1}{4} e^{-\psi} F_{(1)}^2 \right] \tag{2.9}$$

where we have omitted all the terms involving covariant derivatives of the scalars and all components of $\hat{H}_3$ except $\hat{H}_{try} = F_{tr}^{(1)}$, which are assumed to be zero at the horizon. We can repeat the same tricks with the R^4 corrections.

[3] These relations assume (-+++) signature of the metric. Signs in front of e^{ψ} have to be inverted had we adopted the alternative signature. There are also variations in the definition of the dimensionally reduced dilaton too. In [7] for example the dilaton is shifted whereas in [32] it stays unchanged. We will adopt the convention in [7].

JHEP03(2008)043

3. Entropy function formalism

Here we review Sen's entropy formalism for the case of a fundamental string winding a circle, on type IIB supergravity. We are unable to work in ten dimensions, as there are no known regular solutions for this system [25, 33]. However, one can follow Sen and assume that upon compactification, the fundamental string will form an extremal black hole in four dimensions, with a near-horizon geometry of $AdS_2 \times S^2$. This metric can then be lifted to ten dimensions, on $AdS_2 \times S^2 \times T^6$, following the formulae in the previous section.

We consider an extremal black hole solution with near horizon geometry

$$\begin{aligned} ds^2 &= a\left(-r^2dt^2 + \frac{dr^2}{r^2}\right) + b(d\theta^2 + \sin^2\theta d\phi^2) \\ H_{rty} &= e_1, \qquad\qquad F_{rt} = e_2 \\ S &= e^{-2\phi}, \qquad\qquad T = e^{\psi/2} \end{aligned} \tag{3.1}$$

Using this background geometry, we compute the function defined by (2.3). Since we are in the string frame, there is an overall factor of S, and by re-scaling the fields f can be written as

$$\begin{aligned} f &= Sg(a,b,c,d) \\ c &= e_2 T \qquad\qquad d = \frac{e_1}{T} \end{aligned} \tag{3.2}$$

The usual procedure is to extremize f with respect to the moduli and to determine the values of them. One then gets the set of equations

$$\begin{aligned} \frac{\partial f}{\partial S} &= 0 &\leftrightarrow&& g(a,b,c,d) &= 0 \\ \frac{\partial f}{\partial a} &= 0 &\leftrightarrow&& \frac{\partial g}{\partial a} &= 0 \\ \frac{\partial f}{\partial b} &= 0 &\leftrightarrow&& \frac{\partial g}{\partial b} &= 0 \\ \frac{\partial f}{\partial T} &= 0 &\leftrightarrow&& c\frac{\partial g}{\partial c} &= d\frac{\partial g}{\partial d} \end{aligned} \tag{3.3}$$

These equations can then be solved for a, b, c, d. The next step is to compute the Legendre transform of this function, with respect to the variables (e_1, e_2). This is

$$F(a,b,q) \equiv 2\pi(e_i q_i - f(a,b,e_i)), \qquad q_i = \frac{\partial f}{\partial e_i} \tag{3.4}$$

where

$$\begin{aligned} q_1 &= \frac{\partial f}{\partial e_1} \qquad & q_1 &= \frac{S}{T}\frac{\partial g}{\partial d} \\ q_2 &= \frac{\partial f}{\partial e_2} \qquad & q_2 &= ST\frac{\partial g}{\partial c} \end{aligned} \tag{3.5}$$

Using the equations (3.3) and the explicit forms of the q_i's above, it is possible to express S and T as

$$S = \frac{\sqrt{q_1 q_2}}{\sqrt{\frac{\partial g}{\partial c}\frac{\partial g}{\partial d}}} \qquad T = \sqrt{\frac{q_2}{q_1}\frac{\frac{\partial g}{\partial d}}{\frac{\partial g}{\partial c}}} = \sqrt{\frac{q_2}{q_1}\frac{c}{d}} \tag{3.6}$$

Evaluation of the function $F(a, b, q_i)$ yields the value of the entropy. In this case

$$S_{\mathrm{BH}} = 4\pi\sqrt{q_1 q_2}\sqrt{cd} \tag{3.7}$$

The q_i are the charges of the system, and we would like to relate them to the winding number and the Kaluza-Klein momentum of the system. In order to do so, we may use Gauss's Law.

$$e_1 = 4\frac{mR}{16\pi\alpha'}\frac{a}{b}\frac{16\pi G_{10}T^2}{S2\pi RV_5} \qquad e_2 = \frac{n}{4\pi R}\frac{a}{b}\frac{16\pi G_{10}}{2\pi RV_5 ST^2} \tag{3.8}$$

To get the same normalization as in [7], we set $\alpha' = 16$, $\frac{1}{G_4} = \frac{2\pi RV_5}{G_{10}} = \frac{1}{2}$. and the radius of the S^1 to be $R = \sqrt{\alpha'} = 4$. One then gets

$$q_1 = \frac{1}{4}m \qquad q_2 = \frac{1}{4}n \tag{3.9}$$

Finally, the entropy can be expressed in terms of the winding and the Kaluza-Klein momentum. It can then be concluded that in the supergravity approximation, the entropy is zero. This clearly differs from the result obtained from microscopic counting, which is known to be

$$S_{\mathrm{BH}} = 2\pi\sqrt{2nm} \tag{3.10}$$

However, it is a well-known conjecture that one can still get a non-vanishing macroscopic value for the entropy, if one considers higher derivative corrections to the supergravity action [4, 26 – 30]. The horizon area which was initially estimated to be zero, gets "stretched" by quantum/stringy effects. The formalism presented above, can be used to compute the corrections to the entropy.

4. String and brane probes and the no-force conditions

4.1 String probes in AdS background

The supergravity background produced by a fundamental string with winding w and momentum p along a compact circle corresponds to a two-charge black hole. Using Sen's formalism we can solve for the values of the moduli in the near horizon limit of these black holes, while considering the effect of higher order corrections to the supergravity action. These solutions do not necessarily preserve supersymmetry and we need additional checks to identify which ones do. One possibility would be to put in a fundamental string probe in these curved backgrounds [9] and then look for constraints on the background ensuring the vanishing of the force. The resulting condition might then be used as a test for supersymmetry. It is important to note that the curvature of the geometry is of the order of $1/\alpha'$ and so R_{AdS}/α' is not a good expansion parameter. The probe solutions we are looking for have

JHEP03(2008)043

vanishing second-order derivatives. Therefore the Nambu-Goto/DBI action corresponds to a re-sumed series in α'. On the other hand, we are neglecting the backreaction of the probe strings. In the large p, w limit, the effects of back-reaction can be expanded in powers of $1/pw$ [29, 30]. For a probe fundamental string, the backreaction is expected to appear at the next order in $1/pw$. As a result, the constraints that are found using these fundamental string probes constitute a reasonable test for identifying supersymmetric configurations.

There is also an additional set of fundamental strings winding the sphere in the near-horizon $AdS_2 \times S^2$ geometry, which are referred to as dual probes. These are analogous to the dual giants considered in [10, 11]. We will show that these probes also yield solutions satisfying BPS-like conditions.

4.1.1 Probes in Poincaré coordinates

In Poincaré coordinates, the metric lifted to 10 dimensions is given by

$$ds^2 = a\left(-r^2 dt^2 + \frac{dr^2}{r^2}\right) + (crdt + dy)^2 + bd\Omega_2^2 + \sum_{i=1}^{5} dx_i^2, \tag{4.1}$$

where c is proportional to the momentum along the y-direction and one has

$$B_{ty} = dr \tag{4.2}$$

The Lagrangian of a fundamental string winding along y such that $t = \tau, y = \sigma$ is

$$S = \int d^2\sigma \left[\sqrt{-(\dot{X}^2 X'^2 - (\dot{X}\cdot X')^2)} + B_{ty}\right] \tag{4.3}$$

Some useful expressions are

$$\begin{aligned}
\dot{X}^2 &= (c^2 - a)r^2 + a\frac{\dot{r}^2}{r^2} + b\dot{\Omega}_2^2 + \dot{x_i}^2 \\
X'^2 &= 1 + a\frac{r'^2}{r^2} + b\Omega_2'^2 + x_i'^2 \\
\dot{X}\cdot X' &= a\frac{\dot{r}r'}{r^2} + cr + b\dot{\Omega}_2\cdot\Omega_2' + \dot{x}_i\cdot x_i'
\end{aligned} \tag{4.4}$$

The momenta are given by

$$-P_a = g_{ab}\frac{X'^2\dot{x}^b - (\dot{X}\cdot X')x'^b}{\sqrt{-(\dot{X}^2X'^2 - (\dot{X}\cdot X')^2)}} \tag{4.5}$$

For notational simplicity, we write $\sqrt{-(\dot{X}^2X'^2 - (\dot{X}\cdot X')^2)} = \sqrt{-\det G}$. We also use capital X for all ten dimensional coordinates and little x for the coordinates transverse to the string world-sheet, namely $x \in \{r, \theta, \phi, x_i\}, i = 1, 2, \dots 5$. Writing

$$P_\theta\theta' + P_r r' + P_\phi\phi' + P_{x_i}x_i' = P_x x', \tag{4.6}$$

and using

$$\sqrt{-\det G}\left(P_\theta\theta' + P_r r' + P_\phi\phi' + P_{x_i}x_i'\right) - crX'^2 = -\left(\dot{X}\cdot X'\right). \tag{4.7}$$

one can show that

$$(-\det G)\left(P_aP_bg^{ab}+X'^aX'^bg_{ab}+\left(P_xx'\right)^2\right)=ar^2\left(X'^2\right)^2. \qquad (4.8)$$

which then gives a Hamiltonian

$$H=\sqrt{a}r\sqrt{P_aP_bg^{ab}+X'^aX'^bg_{ab}+(P_xx')^2}+crP_xX'+B_{ty} \qquad (4.9)$$

It is observed that the terms inside the square of the Hamiltonian can be re-written as

$$P_aP_bg^{ab}+X'^aX'^bg_{ab}+(P_xX')^2=(1\pm P_x\cdot x')^2+(P\mp x')_a(P\mp x')_bg^{ab}, \qquad (4.10)$$

bearing in mind that $X'^2=y'^2+x'^2=1+x'^2$. A BPS-like solution would then be given by

$$P_a=x'_a. \qquad (4.11)$$

The equations of motion obtained from the Hamiltonian, assuming that all momenta are time independent, are given by

$$\partial_\sigma(\sqrt{a}r+cr)P_a=\frac{\partial H}{\partial x^a} \qquad (4.12)$$

The left hand side is zero for all r if $\sqrt{a}+c=0$. To make the right hand side zero, $\theta=\pi/2$ from the θ equation. In the r equation, the r.h.s. is given by

$$\sqrt{a}(1+P_xx')+cP_xx'+d+\frac{r^2P_r^2}{\sqrt{a}\sqrt{P_aP_bg^{ab}+X'^aX'^bg_{ab}+(P_xX')^2}} \qquad (4.13)$$

which vanishes for $P_r=r'=\sqrt{a}+d=0$ from the r equation. There is then no further constraint on P_ϕ or P_i, and immediately one sees that $|c|=|d|$. Summarizing the result, one finds that the no-force condition for this configuration is given by

$$a=c^2 \qquad c=d \qquad (4.14)$$

Substituting the solution back into the Hamiltonian, one can check that it is identically zero.

4.1.2 Probes in global coordinates

In global coordinates, the metric lifted to 10 dimensions is given by

$$ds^2=-(1+ar^2)dt^2+\frac{dr^2}{(1+ar^2)}+(crdt+dy)^2+bd\Omega_2^2+\sum_{i=1}^{6}dx_i^2 \qquad (4.15)$$

The metric is altered so the equations obtained in the previous section change accordingly. Equations (4.4) become

$$\begin{aligned}\dot{X}^2&=-1+(c^2-a)r^2+\frac{\dot{r}^2}{1+ar^2}+b\dot{\Omega}_2^2+\dot{x}_i{}^2\\ X'^2&=1+\frac{r'^2}{1+ar^2}+b\Omega_2'^2+x_i'^2\\ \dot{X}\cdot X'&=\frac{\dot{r}r'}{1+ar^2}+cr+b\dot{\Omega}_2\cdot\Omega_2'+\dot{x}_i\cdot x_i'\end{aligned} \qquad (4.16)$$

The form of the identity (4.7) remains the same. The Hamiltonian becomes now

$$H_g = \sqrt{1+ar^2}\sqrt{P^2 + X'^2 + (P_x.x')^2} + crP_x x' + dr. \tag{4.17}$$

and the equations of motion are

$$\partial_\sigma\left[\left(\sqrt{1+ar^2} + cr\right)P_x\right] = \frac{\partial H_g}{\partial x}. \tag{4.18}$$

The θ equation again requires $\theta = \pi/2$. The r.h.s of the r equation becomes

$$\begin{aligned}&\frac{ar}{\sqrt{1+ar^2}}(1 + P_x \cdot x') + cP_x x' + d \\ &\quad + \frac{\sqrt{1+ar^2}P_r^2 ar}{\sqrt{1+ar^2}\sqrt{P^2 + X'^2 + (P_x \cdot x')^2}} - \frac{r'^2 ar^2}{2(1+ar^2)^{\frac{3}{2}}\sqrt{P^2 + X'^2 + (P_x \cdot x')^2)}}.\end{aligned} \tag{4.19}$$

A solution would then be given by

$$\begin{aligned} r &= 0 \\ d + cP_x \cdot x' &= 0 \\ \theta &= \frac{\pi}{2} \end{aligned} \tag{4.20}$$

Note here that the probe is stuck at the center of the AdS space. The same property is found in other probe solutions in similar settings [34–36]. The energy of these solutions is

$$H_g = 1 + P_x \cdot x' = 1 - \frac{d}{c}. \tag{4.21}$$

which vanishes identically for $c = d$.

4.2 Dual probes

The derivation of the classical solutions for dual probes can be obtained in exactly the same manner. Notice that in all the solutions that we found, P_θ has to be zero.

4.2.1 Dual probes in Poincaré coordinates

The metric is just as given in (4.1). Wrapping the dual probe about ϕ and following the same procedures as in the previous section, the Hamiltonian is now given by

$$H_{\text{dualp}} = \sqrt{a}r\left[\sqrt{\left(\sqrt{b}\sin\theta \pm \frac{P\cdot x'}{\sqrt{b}\sin\theta}\right)^2 + (P \mp x')^2}\right] + crP_y + dry', \tag{4.22}$$

where now

$$P \cdot x' = P_r r' + P_y y' + P_\theta \theta' + P_{x_i} x_i'. \tag{4.23}$$

A class of solutions is again given by

$$P = x'. \tag{4.24}$$

The θ equations of motion require that $\theta = \pi/2$. The r equation is

$$\dot{P}_r + P_r' = \sqrt{a}\left(\sqrt{b}\sin\theta + \frac{P\cdot x'}{\sqrt{b}\sin\theta}\right) + cP_y + \frac{\sqrt{a}r^2 P_r^2}{\left(\sqrt{b}\sin\theta + \frac{P\cdot x'}{\sqrt{b}\sin\theta}\right)} + dy'. \tag{4.25}$$

For $\sin\theta = 1$, the r.h.s. of the above equation simplifies to

$$\sqrt{ab} + dy' + cP_y + P\cdot x'\sqrt{\frac{a}{b}} = 0. \tag{4.26}$$

The no-force condition when $x' = \dot{x} = 0$ is again

$$\sqrt{b(a-c^2)r} = 0, \tag{4.27}$$

which gives $a = c^2$ at general r, as we had before for the probes.

4.2.2 Dual probes in global coordinates

Again wrapping the string about ϕ, the Hamiltonian is found to be

$$H_{\text{dualg}} = \sqrt{1+ar^2}\left[\sqrt{\left(\sqrt{b}\sin\theta \pm \frac{P\cdot x'}{\sqrt{b}\sin\theta}\right)^2 + (P\mp x')^2}\right] + crP_y + dry' \tag{4.28}$$

The θ equation requires $\theta = \pi/2$ as before. The r equation gives

$$\begin{aligned}\partial_\sigma(\sqrt{1+ar^2}P_r) = &\left[\frac{ar}{\sqrt{1+ar^2}}\left(\sqrt{b}\sin\theta + \frac{P\cdot x'}{\sqrt{b}\sin\theta}\right) + cP_y + dy'\right] \\ &+ \frac{1}{\sqrt{b}\sin\theta + \frac{P\cdot x'}{\sqrt{b}sin\theta}}\left[P_r^2 ar - \frac{r'^2 ar}{(1+ar^2)^2}\right].\end{aligned} \tag{4.29}$$

To make the r.h.s. zero, one demands

$$\begin{aligned} r &= 0 \\ dy' + cP_y &= 0 \end{aligned} \tag{4.30}$$

This gives $c = d$. We see that also the dual probes are stuck in the center of AdS.

4.3 D2 probes in type IIA fundamental string background

In order to account for the entropy from the degeneracy of probe brane solutions, the probes are expected to carry the same quantum number as the background. However, the solutions we have found so far imply relations between the momenta and the winding. As a result, these solutions actually carry less independent charges than the background solution and it is not expected to produce the correct entropy upon quantization. One could use the same techniques as in [35] to account for the entropy, and for this the correct set of probes is required. This shall be considered at length in the next section.

If winding fundamental strings are not carrying the correct quantum numbers of a probe, the logical step would be to find the next simplest candidate. This leads us to consider D2-brane probes in type IIA theory.

JHEP03(2008)043

4.3.1 Poincaré coordinates

Now consider a D2-brane wrapping the S^2 in the same curved background as the one given in (4.1) in section (4.1.1). The Lagrangian is then

$$L = \sqrt{\det G + B + F}, \tag{4.31}$$

where $G + B$ is the pull-back of the spacetime metric and NS 2-form and F is the world-volume electromagnetic field. Expanding the determinant, one can write it as

$$\dot{X}^2(\partial_\theta X)^2(\partial_\phi X)^2 - (\partial_\theta(\dot{X}\cdot\partial_\phi X) - \dot{X}\cdot\partial_\phi X))^2 + (F\dot{X} + K_{[\phi}X_{\theta]})^2, \tag{4.32}$$

where the squares imply implicit contraction with the metric and

$$\begin{aligned} K_i &= F_{ti} + B_{ty}\partial i y = E_i + dr\partial_i y \\ F &= F_{\theta\phi} \end{aligned} \tag{4.33}$$

Now suppose

$$\dot{r} = \partial_i r = 0 \tag{4.34}$$

and that all second derivatives of all the fields vanish. The y-momentum is given by

$$\begin{aligned} LP_y = &(b + (\partial_\theta y)^2)(b\sin^2\theta + (\partial_\phi y)^2)(cr + \dot{y}) \\ &-b((cr+\dot{y})(\partial_\phi y)^2 + \sin^2\theta(cr+\dot{y})(\partial_\theta y)^2) + crF^2 + F\left(F\dot{y} + K_{[\phi}y_{\theta]}\right) \end{aligned} \tag{4.35}$$

All terms involving a total derivative by τ or ϕ are zero automatically. Since θ, as a world-volume coordinate appears explicitly in the action, we must ensure that all terms differentiated by θ vanish. Therefore we need only to focus our attention on setting certain terms to zero in the equations of motion. These terms are given by

$$\frac{\partial L}{\partial F} = 0 \qquad \frac{\partial L}{\partial r} = 0 \qquad \frac{\partial L}{\partial(\partial_\theta y)} = 0. \tag{4.36}$$

Suppose that we let $\partial_\theta y = \partial_\phi y = 0$. We can simplify the equations to

$$b^2\sin^2\theta[-ar + c(cr+\dot{y})] - 2\left(a - c^2\right)rF^2 + 2cF^2\dot{y}^2 = 0 \tag{4.37}$$

$$crFE_\phi + F\dot{y}E_\phi + drE_\theta b\sin^2\theta = 0 \tag{4.38}$$

$$\left(-a + c^2\right)r^2F + crF\dot{y} + crF\dot{y} + F\dot{y}^2 = 0 \tag{4.39}$$

In eq. (4.37) we need to set the piece proportional to $\sin\theta$ and the rest to zero independently. Eqs. (4.37) and (4.39) give then $a = 0$, so there is no sensible solution. However, to obtain non-zero P_y and winding (i.e. non-zero $\partial_i y$), we could equally have set $\dot{y} = 0$, but excite non-trivial electromagnetic waves. Also, since θ is not a periodic coordinate, we can let $\partial_\theta y = 0$. The equations are then reduced to

$$b\left(c^2 - a\right)r = 0 \tag{4.40}$$

$$r(\partial_\phi y)^2\left(c^2 - a\right) - bc^2 r(\partial_\phi y)^2 + db\partial_\phi y + rF^2\left(c^2 - a\right) - E_\theta\partial_\phi y = 0 \tag{4.41}$$

$$E_\theta dr = 0 \tag{4.42}$$

$$K_\phi(crF - E_\theta\partial_\phi y) + drE_\theta(\partial_\phi y)^2 - dFcr^2\partial_\phi y = 0 \tag{4.43}$$

$$r\{(c^2 - a)F - E_\theta\partial_\phi y\} = 0. \tag{4.44}$$

We certainly do not wish $r = 0$ since the momentum would be identically zero. Hence, any non-trivial solution exists only if

$$a = c^2. \tag{4.45}$$

Similarly we have $E_\theta = 0$. Then (4.40), (4.41) and (4.43) are satisfied. From (4.41) we have

$$\partial_\phi y[-\partial_\phi y bc^2 r + db(E_\phi + dr\partial_\phi y)] = 0. \tag{4.46}$$

Also from (4.42) we have

$$crFE_\phi = 0. \tag{4.47}$$

Again we do not want either r or F to vanish for non-trivial momentum. Therefore $E_\phi = 0$. and one gets

$$b\,(\partial_\phi y)^2\left[-c^2 + d^2\right] = 0. \tag{4.48}$$

and we have to conclude that non-trivial winding is only possible if

$$d^2 = c^2. \tag{4.49}$$

Putting the solution into the Lagrangian we are left with $\sqrt{c^2 - d^2} = 0$. This suggests that the Hamiltonian would be proportional to the momenta and thus satisfies a BPS-like condition.

4.4 Comments on probes

The resulting no-force conditions are supposed to hold if the background solution is to represent a supersymmetric configuration. An interesting observation is that the background found in [37] for the heterotic dyonic black hole also satisfies the condition coming from the string probe analysis, which should apply to that case as well. One should also point out that the no-force condition will remain unchanged when considering backgrounds of the form $AdS_2 \times S^1 \times S^n$, with $n \geq 2$.

For the D2 probe, given that its mass scales as $1/g_s$ and the attractor values of g_s in the near horizon limit are proportional to $1/\sqrt{pw}$, one could argue that they are heavy enough to produce significant backreaction on the background. The main purpose of the exercise is to obtain BPS conditions constraining the background, and one could entertain the possibility that the backreaction does not alter the no-force conditions we found. In fact, this is analogous to finding no-force conditions between parallel branes, where is well known that the backreaction does not alter the final result. Since the constraints obtained from the D2-brane probes are identical to those coming from fundamental strings probes, namely $a = c^2$ and $c = d$, we argue that this is further evidence of the condition for supersymmetry, and that backreaction from the probes will not alter the result. Incidentally, the first condition is required if the AdS_2 space were embedded in AdS_3 [12, 38].[4] If the AdS_2

[4]We shall review the details in appendix A.

geometry is indeed embedded in AdS_3, the bulk geometry would possess the conformal symmetry of a two-dimensional CFT, hence giving a holographic understanding of the fundamental string worldsheet CFT. This idea has been recently explored in the literature and various WZW models representing the bulk geometry are have been proposed in [14–16]. In this light, we are lead to conclude that our probe solutions are probably seeing the correct physics. Furthermore, these solutions carry independent winding and momentum charges, and should, upon quantization, be able to account for the black hole entropy.

5. No-force conditions and the WZW action

In the previous section, the no-force constraints we obtained indicate that the background geometry is in fact an AdS_2 embedded in AdS_3. It is therefore natural to consider the fundamental string as a hologram, in which the bulk theory can be constructed as an exact CFT. This idea has been pursued in [14, 15]. The bulk string theory is generally formulated as a supersymmetric $\mathrm{SL}(2,\mathcal{R})$ WZW model, whose level is determined from matching the central charge of the bulk $\mathrm{SL}(2,\mathcal{R})$ current algebra with that of the Virasoro algebra in the boundary CFT by virtue of the AdS/CFT correspondence. Therefore, the type II and heterotic WZW bulk worldsheet theories are distinguished by their number of worldsheet supersymmetries and the central extension of the current algebra. The bulk worldsheet quantum effects, which can be expressed as an expansion in α', together with the effects of the worldsheet fermions, can be taken into account completely such that an effective action can be written down [39]. The aggregate effect is a shift in the level of the $\mathrm{SL}(2,\mathcal{R})$ algebra. It is then possible to consider the bosonic sector of the effective worldsheet action for the type II string, and determine the values of the radii correct, which should be exact to all orders in α'.[5]

To begin with, consider the bosonic sector of a level k WZW action,[6] which is given by

$$S = \frac{k}{4\pi}\int_{\mathcal{M}} d^2z \mathrm{Tr}\left(\partial g^{-1}\bar{\partial} g\right) + \frac{k}{12\pi}\int_{\mathcal{N}} \mathrm{Tr}(\omega^3) \tag{5.1}$$

where $\omega = g^{-1}dg$ is the Maurer-Cartan form. As mentioned before, we will consider the case $k = 2$, in which case the bulk string theory has the correct central charge. Here $g \in \mathrm{SL}(2,R)$ and can be parametrized by

$$g = \begin{pmatrix} z - \frac{\gamma^+\gamma^-}{z} & \frac{\gamma^-}{z} \\ -\frac{\gamma^+}{z} & \frac{1}{z} \end{pmatrix} \tag{5.2}$$

where z, $\gamma^\pm$ are Poincairé coordinates on AdS_3, whose metric is given by

$$ds^2 = \frac{r^2_{AdS_3}}{z^2}(d\gamma^+ d\gamma^- + dz^2) \tag{5.3}$$

[5]The radii of the AdS geometries for the models under consideration is of the order of the string scale. It is not entirely clear whether the geometrical interpretation at such tiny length scales makes sense but this situation is not unexpected for small black holes where *a priori* we knew that the radius of curvature is of the order of the string scale.

[6]Here we consider the effective action, where world-sheet quantum effects and fermions have been accounted for by the value of the level k.

The action can be rewritten in terms of the Polyakov action, with the AdS_3 metric and a Neveu-Schwarz two-form field strength given by

$$H = \frac{q}{r^3}\eta \tag{5.4}$$

and η is the volume form associated to the AdS_3 metric. The Polyakov action reads

$$S = \frac{1}{2\pi\alpha'}\int d^2x G_{\mu\nu}\partial X^\mu \bar{\partial} X^\nu + B_{\mu\nu}\partial X^\mu \bar{\partial} X^\nu \tag{5.5}$$

which can be rewritten in the form of (5.1) as

$$S = \frac{1}{2\pi\alpha'}\left\{\left(\frac{r^2_{AdS_3}}{2}\right)\int d^2z \mathrm{Tr}(\partial g^{-1}\bar{\partial} g) + \frac{q}{12}\int \mathrm{Tr}(\omega^3)\right\} \tag{5.6}$$

From this expression we find that

$$r^2_{AdS_3} = \alpha' k \qquad\qquad q = 2\alpha' k \tag{5.7}$$

It is well-known that AdS_3 can be reduced to $AdS_2 \times S^1$ by using an appropiate choice of coordinate transformations [12]. The metric for $AdS_2 \times S^1$ in Poincairé coordinates is given by

$$ds^2 = a\left(-r^2 dt^2 + \frac{dr^2}{r^2}\right) + (crdt + dy)^2 \tag{5.8}$$

where

$$a = c^2 = \frac{1}{4}r^2_{AdS_3} \tag{5.9}$$

In this set of coordinates, the field strength becomes

$$H_{rty} = d = \frac{q}{r^3_{AdS_3}}a \tag{5.10}$$

Then one immediately sees that

$$d = \sqrt{a} \tag{5.11}$$

Equations (5.9) and (5.11) are the same as the no-force constraints found in section 4. The key point is that these conditions directly result from the symmetry of the WZW model and are independent of α' corrections, and the probes in earlier sections constrain the background to have precisely this symmetry using only no-force arguments.

Setting $\alpha' = 16$ in our convention, and considering that for type II strings, $k = 2$ [14, 15], one gets

$$a = 8 \qquad\qquad c = d = 2\sqrt{2} \tag{5.12}$$

which when substituted into (3.7), reproduces the result in (3.10). This result reinforces the fact that Sen's entropy formalism, which corresponds to the Wald's formula, is consistent with the AdS/CFT construction.

6. Higher derivative corrections

In the previous section we determined the geometry by considering a general WZW model proposed in [14, 15] . However, it is still of interest to explore the effects of higher derivative corrections to the entropy coming from leading α' corrections to the IIB supergravity action and investigate the role played by the no-force constraints and the field redefinition parameters. The correction terms in the action can be determined from string scattering amplitudes [40, 41]. Coefficients for terms proportional to the Ricci tensor remain undetermined, since the scattering amplitudes are evaluated on-shell (to lowest order in α'), and so they vanish. Actions with different linear combinations of these terms are related by field redefinitions [18], since these do not alter the scattering amplitudes. However, while remaining ambiguous, they would contribute to the entropy via Sen's entropy function [42] as discussed earlier. These ambiguities are expected to be canceled out once all corrections from all orders in α' are taken into account. Therefore, it is interesting to see how such ambiguities enter the determination of the entropy at finite order, and if one could find a special field redefinition that gives the full entropy at this order without receiving further corrections. It would also be gratifying to see that higher order corrections at some finite order in α' do give rise to a stretched horizon in general, and to see whether any of the solutions stretching the horizon are supersymmetric at that order.

As a first step we will consider the effect of adding general $\mathcal{R}^4$ terms to the solutions of the attractor equations (3.3). We refer the reader to appendix B for further details. We start by writing down all possible Lorentz scalars that can be constructed from contraction of four Riemann tensors. For $d > 8$, one has 26 linearly independent Lorentz scalars [43]. Many of these terms are set to zero when considering the on-shell supergravity action. However, we will consider a general linear combination of these terms, so the next-to-leading order Lagrangian is written as

$$L_{off-shell} = \frac{1}{8}\zeta(3)(\alpha')^3 \sum_{i=1}^{26} a_i \mathcal{R}_i^4 \tag{6.1}$$

We now use this form of the Lagrangian to construct the corrections to the entropy function in four dimensions. Naively, it would seem like an ill-advised idea, since most of the coefficients are undetermined. However, we will see that many of these contractions give the same answer for symmetric spaces.

To begin with we consider the dimensionally reduced action ignoring the Kaluza-Klein flux and evaluate the Lagrangian with the four-dimensional near horizon metric (2.1). We will then consider more general near-horizon geometries, but still restricting to corrections in the lower-dimensional gravity sector. Finally in section 6.2, we will lift the near horizon $AdS_2 \times S^2$ metric to ten dimensions and evaluate the Lagrangian including the contributions from the NS flux. This is equivalent to considering all the Kaluza-Klein and NS fluxes when dimensionally reducing the action to four dimensions.

JHEP03(2008)043

6.1 $AdS_2 \times S^n$ horizon

Consider a four dimensional black hole with near horizon geometry $AdS_2 \times S^2$

$$ds^2 = a\left(-r^2 dt^2 + \frac{dr^2}{r^2}\right) + b(d\theta^2 + \sin^2\theta d\phi^2) \tag{6.2}$$

The entropy function receives higher derivative contributions of the form

$$f(a,b,c,d) = \frac{1}{4}abS\left[-\frac{1}{a} + \frac{1}{b} + \frac{c^2}{4a^2} + \frac{d^2}{4a^2}\right] + \frac{1}{8}\zeta(3)(\alpha')^3 S\left[\sum_{i=1}^{26} a_i \mathcal{R}_i^4(a,b,c,d)\right] \tag{6.3}$$

Given that one is free to redefine the fields, and given that some contractions yield the same dependence on the radii, it is simpler to reduce the number of field redefinition parameters to perform the analysis. This is described in more detail in the appendix. Here it suffices to say that the corrections will only include four undefined parameters that remain arbitrary and can be chosen in any way.

Using then the lagrangian (6.1), one can obtain the attractor equations (3.3). The equation $c\frac{\partial g}{\partial c} = d\frac{\partial g}{\partial d}$ will immediately yield $c = d$. The other equations are more complicated, and therefore are included in the appendix. We can restrict our search to supersymmetric configurations by imposing the no-force condition we derived from the probe analysis, namely, $a = c^2$ and $c = d$, the latter of which has been automatically implemented by the equations of motion. The system we are left with is over-determined for given values of the parameters and in general has *no* solution, which suggests that the no-force condition is *incompatible* with the attractor equations obtained from the truncated action excluding flux terms. This strongly indicates that the truncation is inconsistent with supersymmetry.

One could consider more general horizon geometries. We will keep the AdS_2 factor given that the black hole is supposed to be extremal, and allow the dimension of the sphere n to lie between 3 and 8, and proceed in the same manner as before. Again, the number of independent scalars formed with four Riemann tensors is 26, and once more we will ignore corrections involving the Kaluza-Klein and NS fluxes. It is straightforward to compute the contractions, given that we are dealing only with the gravity sector and the geometry is that of maximally symmetric spaces. The analysis proceeds as before, and one finds that the resulting attractor equations are again inconsistent with the no-force conditions. Ignoring issues of consistency for the moment, we could try to look for special regions in the space of redefinition parameters where the no-force conditions and the attractor equations are actually consistent with each other. This is achieved by treating the field redefinition parameters as variables. Let us first eliminate two parameters, so that we are left with an equation whose dependence on the remaining ones drops out

$$\begin{aligned}\frac{1}{a^2bn(n-1)(n-2)}\big[&3a^6n^2 + 3a^6n^4 - 6a^6n^3 + 2a^4bc^2n^2 - 2a^4bc^2n + 36ab^5\\ &-18a^2b^4n^2 - 12b^5c^2 - 6a^5bn^2 + 6a^5bn + 18a^2b^4n\big] = 0\end{aligned} \tag{6.4}$$

Let us now impose the no-force conditions and find solutions for a in terms of b. The only real solution to the system is given by

$$a = \frac{4}{3}\frac{b}{n(n-1)} \tag{6.5}$$

with the other roots being imaginary. The other remaining pair of equations can be used to fix b in terms of the parameters, and solutions could be found explicitly by tuning the parameters. However, we should stress that it seems to be inconsistent to use the no-force conditions when truncating the action to the gravity sector, so let us now consider the effects of using the no-force constraints when taking into account the fluxes.

6.2 Adding the fluxes

In the previous subsection we concluded that the gravity sector alone in general does not stretch the horizon, unless one restricts the field redefinition parameters to a special subspace. It is unclear whether the procedure is consistent. Given that we have neglected the flux terms in the first place, one should after all look for solutions in their presence. In principle, this makes the calculation much more involved, and one could hope to find a solution only if there were a simplification upon the use of the no-force condition.

We start by lifting the near-horizon $AdS_2 \times S^2$ metric to ten dimensions as in (4.1). The resulting geometry is that of $AdS_2 \times S^2 \times S^1 \times T^5$. In order to take the NS fluxes into account, we will consider the conjecture in [23, 24], which gives a prescription for the eight derivative terms in the action, involving the NS-NS form. It should be noted that there are further corrections involving fluxes to the tree-level NS-NS sector at the α'^3 order [44]. For instance, terms of the form $\mathcal{R}^3 H^2$ are present, but are not reproduced when writing the NS-NS form as a torsion.[7] Indeed, the prescription we use here is only valid for the four-point contribution [45] to the effective action, but nevertheless, the major simplification that follows indicates that these terms are consistent with supersymmetry.

The way to implement this conjecture, is by defining an effective Riemann curvature such that the flux terms are packaged in certain combinations carrying the symmetries of the Riemann tensor. Since the derivatives of all the fields are assumed to vanish in the near horizon limit, the surviving terms are given by

$$\tilde{R}_{rs}{}^{pq} = R_{rs}{}^{pq} + \nabla_{[r} H_{s]}{}^{pq} - H_{[r}{}^{u[p} H_{s]u}{}^{q]} \tag{6.6}$$

Corrections at order α'^3 are then built by contracting this generalized Riemann curvature. The first term we shall consider, is the well-known C^4 Weyl curvature term. From the superspace formalism of type IIB string theory [46 − 48], one can show that the combination of eight derivative terms, that will contribute to the action at this order is given by

$$\tilde{C}^4 = -\frac{1}{4}\tilde{C}_{pqrs}\tilde{C}_{pq}{}^{tu}\tilde{C}_{rt}{}^{vw}\tilde{C}_{suvw} + \tilde{C}^{pqrs}\tilde{C}_{p}{}^{t}{}_{r}{}^{u}\tilde{C}_{t}{}^{v}{}_{q}{}^{w}\tilde{C}_{uvsw}, \tag{6.7}$$

where $\tilde{C}$ is the generalised Weyl tensor obtained from $\tilde{R}$. The action can now be evaluated for the metric (4.1) and NS field (4.2). These terms will give rise to all the Kaluza-Klein fields that will contribute to the entropy function of the four dimensional black hole. We will not attempt to write down the explicit form of the entropy function or of the attractor equations for this case, given their formidable length, but in principle one could try to solve them numerically. Instead of proceeding in this way, we will impose the no-force conditions found in section 4, as mentioned above.

[7] We thank M. B. Green, P. Vanhove and D. Tsimpis for their remarks on this issue.

In the previous cases, we saw that the fourth equation in (3.3) always implied the condition $c = d$. This is no longer true, given that the additional terms, do not preserve the symmetry between c and d. However from the probe analysis, we found that for the existence of a supersymmetric solution, the background was required to satisfy $c = d$ in addition to $a = c^2$. Remarkably, by demanding these conditions, one finds that the fourth attractor equation in (3.3) is automatically satisfied, whereas the remaining equations are simplified tremendously.

At this point, we are left in principle with three equations and two unknowns, namely $\{b, d\}$. However it turns out that the first two equations become degenerate, so one has two equations and two unknowns that can be solved numerically. These are

$$54a^4b^3-27a^3b^4-40\zeta(3)ab^3-320\zeta(3)a^3b+240\zeta(3)a^2b^2+35\zeta(3)b^4+4480\zeta(3)a^4=0$$
$$27a^3b^4-640\zeta(3)a^3b-35\zeta(3)b^4+240\zeta(3)a^2b^2+13440\zeta(3)a^4=0 \quad (6.8)$$

One can check that $\{a = 1.15093, b = -23.11745\}$ is a numerical root to this system. However, this solution has negative b, so the signature of the horizon is changed. It was pointed out to us that a change in signature might represent a non-geometric background, and therefore should not be regarded as being physical[8] so we are left with the conclusion that there is no supersymmetric solution that can be found with the C^4 Weyl curvature term.

We could still make the same arguments as before, and look for more generic eight derivative terms. Bearing in mind that perturbative string calculations do not determine the coefficients of these terms uniquely, there is no reason why they should be excluded *a priori*. Therefore, we can proceed as in the previous subsections and consider a linear combination of all 26 $\mathcal{R}^4$ scalars, evaluating this for the generalized Riemann curvature so to include the fluxes. The equations are of course, much more complicated, but fortunately it is still true that by imposing the no-force conditions above, the last equation in (3.3) is satisfied, and the first two equations become identical. Hence, unlike the previous case where the corrections from the fluxes are ignored, one is *always* left with two equations relating d and b for any choice of field-redefinition parameters, and we see that in this case the truncation of the action is *consistent* with the no-force constraints. However, there is no guarantee that the solutions obtained from the present truncation are physical, as we showed explicitly for the Weyl curvature correction term. The value of the entropy will depend on the arbitrary parameters and the dependence is not expected to drop out at any finite order in α'. Our analysis in this section supports the special status of the no-force condition and reinforces the need to study the system using an exact CFT.

7. Conclusions

In this paper we have studied the stretching of the horizon of the type II fundamental string small black hole, and analysed the conditions for preserving supersymmetry. One issue we addressed was how to identify supersymmetric configurations after taking into account higher derivative terms in the action. Given that the α' corrected Killing spinor equations

[8] We thank A. Castro for useful discussions regarding this issue

are yet to be determined, one needs an alternative formalism for identifying supersymmetric configurations. Our procedure has been to probe the background with strings and branes, and find constraints that lead to a vanishing force. It is well known that extremal four dimensional black holes have $AdS_2 \times S^2$ as their near horizon geometry. It has, in the past been assumed that dimensionally reduced small black holes arising from wrapped strings also have this geometry in the presence of α' corrections. Therefore, we considered solutions of probe strings/branes moving in an $AdS_2 \times S^2$ background, and determined constraints relating the radius of the AdS_2 to the fluxes. The "no-force" condition $a = c^2$ we found coincides with the condition that enhances the AdS_2 symmetry to AdS_3.

Motivated by this fact we then looked at the formulation of the bulk geometry in terms of an $\mathrm{SL}(2, \mathcal{R})$ WZW model which takes into account all α' corrections. We determined the values of the moduli by considering the models constructed in [14, 15] and evaluated the entropy using eq. (3.7), which matches the result obtained from microscopic counting. The WZW action also implies both no-force conditions we determined using the probes. This gives further evidence supporting the level two $\mathrm{SL}(2, \mathcal{R})$ WZW model as the holographic dual to the fundamental string worldsheet. It is however surprising that the probe analysis does not seem to receive any higher derivative corrections, contrary to naive expectations.

We also considered higher derivative corrections to the horizon employing the entropy function formalism while imposing the no-force conditions on the background. We showed that upon inclusion of all possible R^4 terms and ignoring corrections to the NS and Kaluza-Klein fluxes, the equations are inconsistent with supersymmetry.

Therefore we then studied the role played by the corrections involving the fluxes in stretching the horizon. The fluxes are taken into account by lifting the near horizon metric to ten dimensions and using the prescriptions in [23, 24]. We showed that the no-force conditions are automatically consistent with the attractor equations for any field redefinition. This resulted from the tremendous simplification obtained after imposing them. Solutions of the attractor equations and subsequently the entropy depend, however, on the ambiguous field redefinition parameters. In particular, motivated by on-shell superspace arguments, we studied corrections taking the form of the well-known C^4 Weyl curvature term. We found a numerical root to the attractor equations where the signature of the near-horizon geometry is changed. We conclude that there are no physically acceptable solutions for this particular choice of field redefinition parameters, but others will in general give solutions in which the horizon is stretched.

It is clear from our analysis that it is unlikely that higher derivative corrections at finite order can possibly reproduce the full microscopic entropy of the type II small black holes. This is different from the heterotic string where the microscopic entropy can be obtained by including only the four-derivative corrections, namely the Gauss-Bonnet term [5]. It has been argued that this is the case because, in the heterotic case, there are non-renormalisation theorems at work. This is justified by studying the same black hole in five dimensions and assuming that the near horizon geometry has an AdS_3 factor. The anomalies of the boundary CFT can be determined uniquely from certain terms in the bulk, which in turn fix the value of the entropy [49, 50]. The same type of argument fails in type II theories given the absence of anomalies, which suggests that one must consider

JHEP03(2008)043

all α' corrections to the supergravity action. This is also consistent with the fact that the field redefinition parameters do not drop out, at least at this order and possibly at any finite order in α'. The WZW formulation of the bulk geometry is probably the best handle in understanding these small black holes.

A. AdS_2 and AdS_3

Here we show explicitely how to reduce the AdS_3 metric to $AdS_2 \times S^1$, following [12].

Consider the AdS_3 metric in Poincaré coordinates.

$$ds_3^2 = \frac{1}{y^2}(dw^+ dw^- + dy^2). \tag{A.1}$$

Now apply the following transformations

$$\begin{aligned} w^- &= t^-, \\ w^+ &= \frac{1}{2}\exp(2u), \\ y &= \sqrt{\frac{t^+ - t^-}{T}}\exp(u), \end{aligned} \tag{A.2}$$

for some real parameter T. The resultant metric is then

$$ds_3^2 = \frac{(dt^+ - dt^-)^2}{4(t^+ - t^-)^2} + \frac{du(dt^+ + dt^-)}{(t^+ - t^-)} + du^2. \tag{A.3}$$

Rewriting $2t = t^+ + t^-$ and $2x = t^+ - t^-$, the metric becomes

$$ds_3^2 = \left(\frac{-dt^2 + dx^2}{4x^2}\right) + \left(\frac{dt}{2x} + du\right)^2. \tag{A.4}$$

This metric is then identified with that of $AdS_2 \times S^1$, with $a = c^2$ using the parametrisation given in section (4.1.1).

B. $\mathcal{R}^4$ corrections

In this appendix we include some of the details regarding the computation of $\mathcal{R}^4$ corrections to the entropy of the type II fundamental string black hole.

General $\mathcal{R}^4$ terms, for $d > 8$, can be expressed in terms of a basis of 26 independent Lorentz scalars built from contractions of four Riemann tensors. A particular basis for these is reproduced below [43]

$$\begin{aligned}
\mathcal{R}_1^4 &= R^4 & \mathcal{R}_{14}^4 &= R^{pq}R^{rs}R^{t\ \ u}_{\ p\ \ r}R_{tqus} \\
\mathcal{R}_2^4 &= R^2 R^{pq}R_{pq} & \mathcal{R}_{15}^4 &= RR^{pqrs}R_{pq}^{\ \ \ tu}R_{rstu} \\
\mathcal{R}_3^4 &= RR^{pq}R_p^{\ r}R_{qr} & \mathcal{R}_{16}^4 &= RR^{pqrs}R_{p\ r}^{\ t\ u}R_{qtsu} \\
\mathcal{R}_4^4 &= (R^{pq}R_{pq})^2 & \mathcal{R}_{17}^4 &= R^{pq}R_{p\ q}^{\ r\ s}R^{tuv}_{\ \ \ r}R_{tuvs} \\
\mathcal{R}_5^4 &= R^{pq}R_p^{\ r}R_q^{\ s}R_{rs} & \mathcal{R}_{18}^4 &= R^{pq}R^{rstu}R_{rs\ p}^{\ \ v}R_{tuvq}
\end{aligned}$$

$$
\begin{aligned}
\mathcal{R}^4_6 &= RR^{pq}R^{rs}R_{prqs} & \mathcal{R}^4_{19} &= R^{pq}R^{rstu}R_r{}^v{}_{tp}R_{svuq} \\
\mathcal{R}^4_7 &= R^{pq}R^{rs}R_r{}^tR_{psqt} & \mathcal{R}^4_{20} &= (R^{pqrs}R_{pqrs})^2 \\
\mathcal{R}^4_8 &= R^2R^{pqrs}R_{pqrs} & \mathcal{R}^4_{21} &= R^{pqrs}R^{pqrt}R^{uvw}{}_sR_{uvwt} \\
\mathcal{R}^4_9 &= RR^{pq}R^{rst}{}_pR_{rstq} & \mathcal{R}^4_{22} &= R^{pqrs}R_{pq}{}^{tu}R_{tu}{}^{vw}R_{rsvw} \\
\mathcal{R}^4_{10} &= R^{pq}R_{pq}R^{rstu}R_{rstu} & \mathcal{R}^4_{23} &= R^{pqrs}R_{pq}{}^{tu}R_{rt}{}^{vw}R_{suvw} \\
\mathcal{R}^4_{11} &= R^{pq}R_p{}^rR^{stu}{}_qR_{stur} & \mathcal{R}^4_{24} &= R^{pqrs}R_{pq}{}^{tu}R_r{}^v{}_t{}^wR_{svuw} \\
\mathcal{R}^4_{12} &= R^{pq}R^{rs}R^{tu}{}_{pr}R_{tuqs} & \mathcal{R}^4_{25} &= R^{pqrs}R_p{}^t{}_r{}^uR_t{}^v{}_u{}^wR_{qvsw} \\
\mathcal{R}^4_{13} &= R^{pq}R^{rs}R^t{}_p{}^u{}_qR_{trus} & \mathcal{R}^4_{26} &= R^{pqrs}R_p{}^t{}_r{}^uR_t{}^v{}_q{}^wR_{uvsw}
\end{aligned}
$$

Corrections to the supergravity lagrangian can be computed by considering linear combinations of the above $\mathcal{R}^4$ terms. Let us first evaluate the corrections coming from the gravitational sector only. This is, one ignores contributions coming from the fluxes. One can check that all the contractions are proportional to six different combinations

$$
\begin{aligned}
\mathcal{R}^4_1 &\sim \frac{(a-b)^4}{a^4b^4} \\
\mathcal{R}^4_2, \mathcal{R}^4_8 &\sim \frac{(a-b)^2(a^2+b^2)}{a^4b^4} \\
\mathcal{R}^4_3, \mathcal{R}^4_6, \mathcal{R}^4_9, \mathcal{R}^4_{15} &\sim \frac{(a-b)(a^3-b^3)}{a^4b^4} \\
\mathcal{R}^4_4, \mathcal{R}^4_{10}, \mathcal{R}^4_{20} &\sim \frac{(a^2+b^2)^2}{a^4b^4} \\
\mathcal{R}^4_5, \mathcal{R}^4_7, \mathcal{R}^4_{11}, \mathcal{R}^4_{12}, \mathcal{R}^4_{13}, \mathcal{R}^4_{14}, \mathcal{R}^4_{17}, \mathcal{R}^4_{18}, \mathcal{R}^4_{21}, \mathcal{R}^4_{22}, \mathcal{R}^4_{23}, \mathcal{R}^4_{25}, \mathcal{R}^4_{26} &\sim \frac{(a^4+b^4)}{a^4b^4} \\
\mathcal{R}^4_{16}, \mathcal{R}^4_{19}, \mathcal{R}^4_{24} &= 0
\end{aligned}
$$

Given that one is free to redefine the fields, we consider a generic linear combination (eq. (6.1)) and group terms that are proportional to the same function of (a, b), the radii of the geometry. For instance, $\mathcal{R}^4_2$ and $\mathcal{R}^4_8$ give the same result, up to a factor of 2, so we set

$$
\begin{aligned}
a_2\mathcal{R}^4_2 + a_8\mathcal{R}^4_8 &= (a_2 + 2a_8)\mathcal{R}^4_2 \\
&= \tilde{a}_2\mathcal{R}^4_2
\end{aligned}
$$

One can proceed analogously with the remaining terms. In the end, one is left with five parameters, namely, $\tilde{a}_1$, $\tilde{a}_2$, $\tilde{a}_3$, $\tilde{a}_4$ and $\tilde{a}_5$. The last parameter, $\tilde{a}_5$, is fixed from an on-shell string amplitude calculation [41]. The rest remain arbitrary. One can now obtain the attractor equations (3.3). The last equation immediately gives $c = d$. Substituing this in the first three equations yields the expressions

$$
\begin{aligned}
-a^3b^4+a^4b^3+\frac{1}{2}a^2b^4c^2+\zeta(3)\big[&22528a_3a^4 + 10240a_2a^4 + 4096a_1a^4 - 16384a_1b^3a \\
&-16384a_1ba^3+24576a_1b^2a^2+43008a_4b^2a^2-20480a_2b^3a+20480a_2b^2a^2 \\
&-20480a_2ba^3-22528a_3b^3a-22528a_3ba^3+21504a_4b^4+21504a_4a^4 \\
&+68608a_5b^4+68608a_5a^4+22528a_3b^4+10240a_2b^4+4096a_1b^4\big] = 0
\end{aligned} \tag{B.1}
$$

$$
\begin{aligned}
a^4b^3-\frac{1}{2}a^2b^4c^2+\zeta(3)\big[&-24576a_1b^2a^2+32768a_1b^3a-20480a_2b^2a^2+40960a_2b^3a \\
&-43008a_4b^2a^2+45056a_3b^3a-205824a_5b^4+21504a_4a^4-64512a_4b^4+4096a_1a^4 \\
&+10240a_2a^4-30720a_2b^4-67584a_3b^4+68608a_5a^4-12288a_1b^4+22528a_3a^4\big]=0
\end{aligned} \tag{B.2}
$$

$$
\begin{aligned}
a^3b^4-\frac{1}{2}a^2b^4c^2+\zeta(3)\big[&24576a_1b^2a^2-32768a_1ba^3+20480a_2b^2a^2+43008a_4b^2a^2 \\
&-45056a_3ba^3-40960a_2ba^3-68608a_5b^4+64512a_4a^4-21504a_4b^4+12288a_1a^4 \\
&+30720a_2a^4-10240a_2b^4-22528a_3b^4+205824a_5a^4-4096a_1b^4+67584a_3a^4\big]=0
\end{aligned} \tag{B.3}
$$

where we have dropped the tilde to avoid cluttering.

More general geometries can also be considered. For horizon geometries of the form $AdS_2 \times S^n$, with the dimension of the sphere between 3 and 8, the $\mathcal{R}^4$ contractions are again easy to compute

$$
\begin{aligned}
\mathcal{R}_1^4 &\sim \left(\frac{n(n-1)}{b}-\frac{2}{a}\right)^4 \\
\mathcal{R}_2^4, \mathcal{R}_9^4 &\sim \left(\frac{n(n-1)}{b}-\frac{2}{a}\right)^2 \left(\frac{n(n-1)^2}{b^2}+\frac{2}{a^2}\right) \\
\mathcal{R}_3^4, \mathcal{R}_6^4 &\sim \left(\frac{n(n-1)}{b}-\frac{2}{a}\right) \left(\frac{n(n-1)^3}{b^3}-\frac{2}{a^3}\right) \\
\mathcal{R}_4^4 &\sim \left(\frac{n(n-1)^2}{b^2}+\frac{2}{a^2}\right)^2 \\
\mathcal{R}_5^4, \mathcal{R}_7^4 &\sim \left(\frac{n(n-1)^4}{b^4}+\frac{2}{a^4}\right) \\
\mathcal{R}_8^4 &\sim \left(\frac{n(n-1)}{b}-\frac{2}{a}\right)^2 \left(\frac{n(n-1)}{b^2}+\frac{2}{a^2}\right) \\
\mathcal{R}_{10}^4 &\sim \left(\frac{n(n-1)}{b^2}+\frac{2}{a^2}\right) \left(\frac{n(n-1)^2}{b^2}+\frac{2}{a^2}\right) \\
\mathcal{R}_{11}^4, \mathcal{R}_{12}^4, \mathcal{R}_{17}^4, &\sim \left(\frac{n(n-1)^3}{b^4}+\frac{2}{a^4}\right) \\
\mathcal{R}_{14}^4 &\sim \left((n^2+n(n-2))\frac{(n-1)^2}{b^4}+\frac{4}{a^4}\right) \\
\mathcal{R}_{15}^4 &\sim \left(\frac{n(n-1)}{b}-\frac{2}{a}\right) \left(\frac{n(n-1)}{b^3}-\frac{2}{a^3}\right) \\
\mathcal{R}_{16}^4, \mathcal{R}_{19}^4, \mathcal{R}_{24}^4 &\sim \frac{1}{b^4} \\
\mathcal{R}_{18}^4, \mathcal{R}_{21}^4 &\sim \left(\frac{n(n-1)^2}{b^4}+\frac{2}{a^4}\right) \\
\mathcal{R}_{20}^4 &\sim \left(\frac{n(n-1)}{b^2}+\frac{2}{a^2}\right)^2 \\
\mathcal{R}_{22}^4, \mathcal{R}_{23}^4 &\sim \left(\frac{n(n-1)}{b^4}+\frac{2}{a^4}\right)
\end{aligned}
$$

$$\mathcal{R}^4_{25} \sim \frac{n^2 + 2n(n-2) + n^2(n-2)^2}{b^4} + \frac{4}{a^4}$$
$$\mathcal{R}^4_{26} \sim \frac{n + 2n^2(n-2) + n(n-2)^2}{b^4} + \frac{2}{a^4}$$
$$\mathcal{R}^4_{13} \sim (n + n^2(n-2))\frac{(n-1)^2}{b^4} + \frac{2}{a^4}$$

These formulae reproduce the table we had before for $n = 2$. As before, some contractions give the same functional dependence on the radii, so it is possible to choose the parameters, such that one is left with a reduced subset of them. We will not reproduce the attractor equations (3.3) for this case, given their length. However, we should say that the behaviour of the system is analogous to the $n = 2$ case.

Corrections involving the fluxes, can be computed in ten dimensions once the action is known. As it is expected, the attractor equations are formidably lengthy and complicated, so we refrain from reproducing them explicitely.

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JHEP03(2008)043

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